Intermodulation Distortion

The 1 dB compression curve shows that amplifiers are only **approximately** linear.

Actually, this should be **obvious**, as amplifiers are constructed with transistors—**non-linear** devices!

So, instead of the **ideal** case:

$$v_{out} = A_v \, v_{in}$$

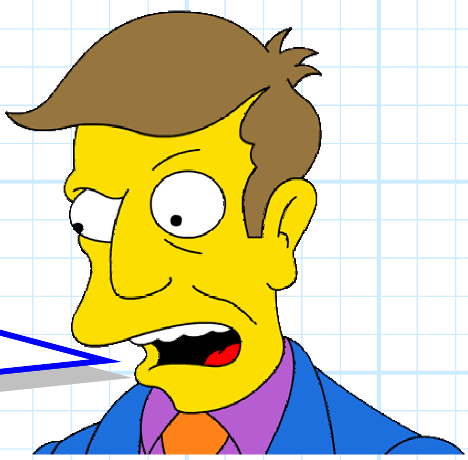
Actual amplifier behavior requires more terms to describe!

$$\boxed{v_{out} = A_v \, v_{in} + B \, v_{in}^2 + C \, v_{in}^3 + \cdots}$$

This representation is simply a **Taylor Series** representation of the **non-linear** function:

$$v_{out} = f(v_{in})$$

Q: ***Non****-linear! But I thought an amplifier was a* ***linear*** *device? After all, we characterized it with a* ***scattering matrix****!*

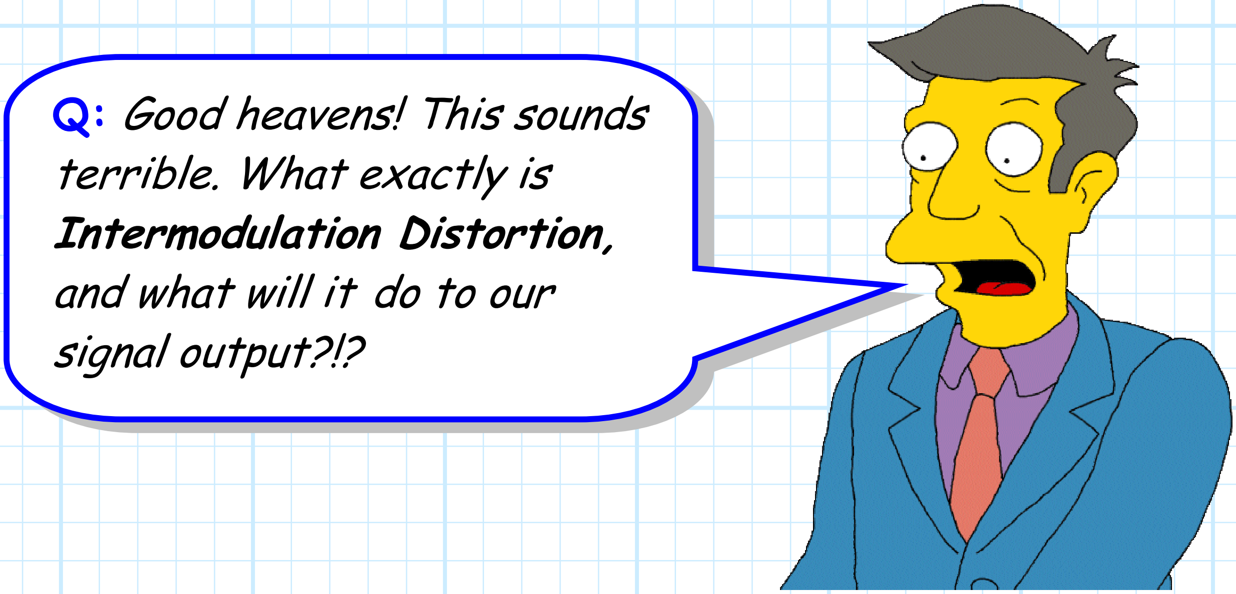


A: Generally speaking, the constants B, C, D, etc. are **very** small compared to the voltage gain A_v. Therefore, **if** v_{in} is likewise small, we can **truncate** the Taylor Series and **approximate** amplifier behavior as the linear function:

$$v_{out} \approx A_v\, v_{in}$$

BUT, as v_{in} gets large, the values v_{in}^2 and v_{in}^3 will get **really** large! In that case, the terms $B\, v_{in}^2$ and $C\, v_{in}^3$ will become **significant**.

As a result, the output will not simply be a larger version of the input. The output will instead be **distorted**—a phenomenon known as **Intermodulation Distortion.**



A: Say the input to the amplifier is sinusoidal, with magnitude a:

$$v_{in} = a\cos\omega t$$

Using our knowledge of trigonometry, we can determine the result of the second term of the output Taylor series:

$$\begin{aligned} B\, v_{in}^2 &= B\, a^2 \cos^2 \omega t \\ &= \frac{B\, a^2}{2} + \frac{B\, a^2}{2} \cos 2\omega t \end{aligned}$$

We have created a **harmonic** of the input signal!

In other words, the input signal is at a frequency ω, while the output includes a signal at **twice** that frequency (2ω).

We call this signal a **second** order product, as it is a result of **squaring** the input signal.

Note we also have a **cubed** term in the output signal equation:

$$v_{out} = A_v\, v_{in} + B\, v_{in}^2 + C\, v_{in}^3 + \cdots$$

Using a trig identity, we find that:

$$\begin{aligned} C\, v_{in}^3 &= C\, a^3 \cos^3 \omega t \\ &= \frac{C\, a^3}{2} \cos \omega t + \frac{C\, a^3}{4} \cos 3\omega t \end{aligned}$$

Now we have produced a **second harmonic** (i.e., 3ω)!

As you might expect, we call this harmonic signal a **third-order** product (since it's produced from v_{in}^3).

Q: *I confess that I am still a bit* ***befuddled****. You said that values B and C are typically* ***much*** *smaller that that of voltage gain A_v. Therefore it would seem that these harmonic signals would be* ***tiny*** *compared to the fundamental output signal $A_v\, a \cos \omega t$. Thus, I* ***don't*** *why there's a problem!*

To understand why intermodulation distortion can be a problem in amplifiers, we need to consider the **power** of the output signals.

We know that the power of a sinusoidal signal is proportional to its **magnitude squared**. Thus, we find that the power of each output signal is related to the input signal power as:

$$\text{1rst-order output power} \doteq P_1^{out} = A_v^2\, P_{in} = G\, P_{in}$$

$$\text{2nd-order output power} \doteq P_2^{out} = \frac{B^2}{4} P_{in}^2 = G_2\, P_{in}^2$$

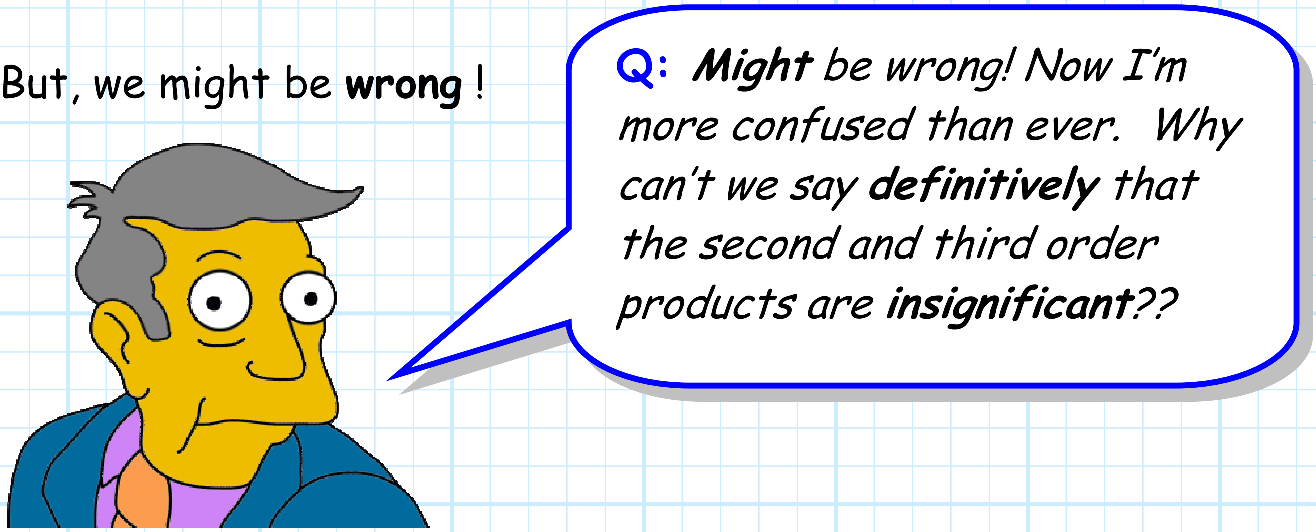
$$\text{3rd-order output power} \doteq P_3^{out} = \frac{C^2}{16} P_{in}^3 = G_3\, P_{in}^3$$

where we have obviously **defined** $G_2 \doteq B^2/4$ and $G_3 \doteq C^2/16$.

Note that unlike G, the values G_2 and G_3 are not coefficients (i.e., not unitless!). The value G_2 obviously has units of inverse power (e.g., mW^{-1} or W^{-1}), while G_3 has units of inverse power squared (e.g., mW^{-2} or W^{-2}).

We know that typically, G_2 and G_3 are much **smaller** than G. Thus, we are **tempted** to say that P_1^{out} is much **larger** than P_2^{out} or P_3^{out}.

But, we might be **wrong** !



Look **closely** at the expressions for the output power of the first, second, and third order products:

$$
\begin{aligned}
P_1^{out} &= G\ P_{in} \\
P_2^{out} &= G_2\ P_{in}^2 \\
P_3^{out} &= G_3\ P_{in}^3
\end{aligned}
$$

This **first** order output power is of course **directly** proportional to the input power. However, the **second** order output power is proportional to the input power **squared**, while the **third** order output is proportional to the input power **cubed**!

Thus we find that if the input power is **small**, the second and third order products **are** insignificant. But, as the input power **increases**, the second and third order products get **big** in a hurry!

For example, if we **double** the input power, the **first** order signal will of course likewise **double**. However, the **second** order power will **quadruple**, while the **third** order power will increase **8 times**.

For **large** input powers, the second and third order output products can in fact be **almost as large** as the first order signal!

Perhaps this can be most easily seen by expressing the above equations in **decibels,** e.g.,:

$$
\begin{aligned}
P_1^{out}(dBm) &= G(dB) + P_{in}(dBm) \\
P_2^{out}(dBm) &= G_2(dBm^{-1}) + 2\left[P_{in}(dBm)\right] \\
P_3^{out}(dBm) &= G_3(dBm^{-2}) + 3\left[P_{in}(dBm)\right]
\end{aligned}
$$

where we have used the fact that $\log x^n = n \log x$. Likewise, we have defined:

$$
\begin{aligned}
G_2(dBm^{-1}) &= 10\log_{10}\left[\frac{G_2}{\left(1/1.0mW\right)}\right] \\
&= 10\log_{10}\left[G_2\left(1.0mW\right)\right]
\end{aligned}
$$

and:

$$
\begin{aligned}
G_3(dBm^{-2}) &= 10\log_{10}\left[\frac{G_3}{\left(1/1.0mW^2\right)}\right] \\
&= 10\log_{10}\left[G_3\left(1.0mW^2\right)\right]
\end{aligned}
$$

Hint: Just express everything in milliwatts!

Note the value $2\left[P_{in}(dBm)\right]$ does **not** mean the value $2P_{in}$ expressed in decibels. The value $2\left[P_{in}(dBm)\right]$ is fact the value of P_{in} expressed in decibels—**times two**!

For **example**, if $P_{in}(dBm) = -30\,dBm$, then $2\left[P_{in}(dBm)\right] = -60\,dBm$. Likewise, if $P_{in}(dBm) = 20\,dBm$, then $2\left[P_{in}(dBm)\right] = 40\,dBm$.

What this means is that for every **1dB** increase in **input** power P_{in} the fundamental (**first-order**) signal will increase **1dB**; the **second-order** power will increase **2dB**; and the **third-order** power will increase **3dB**.

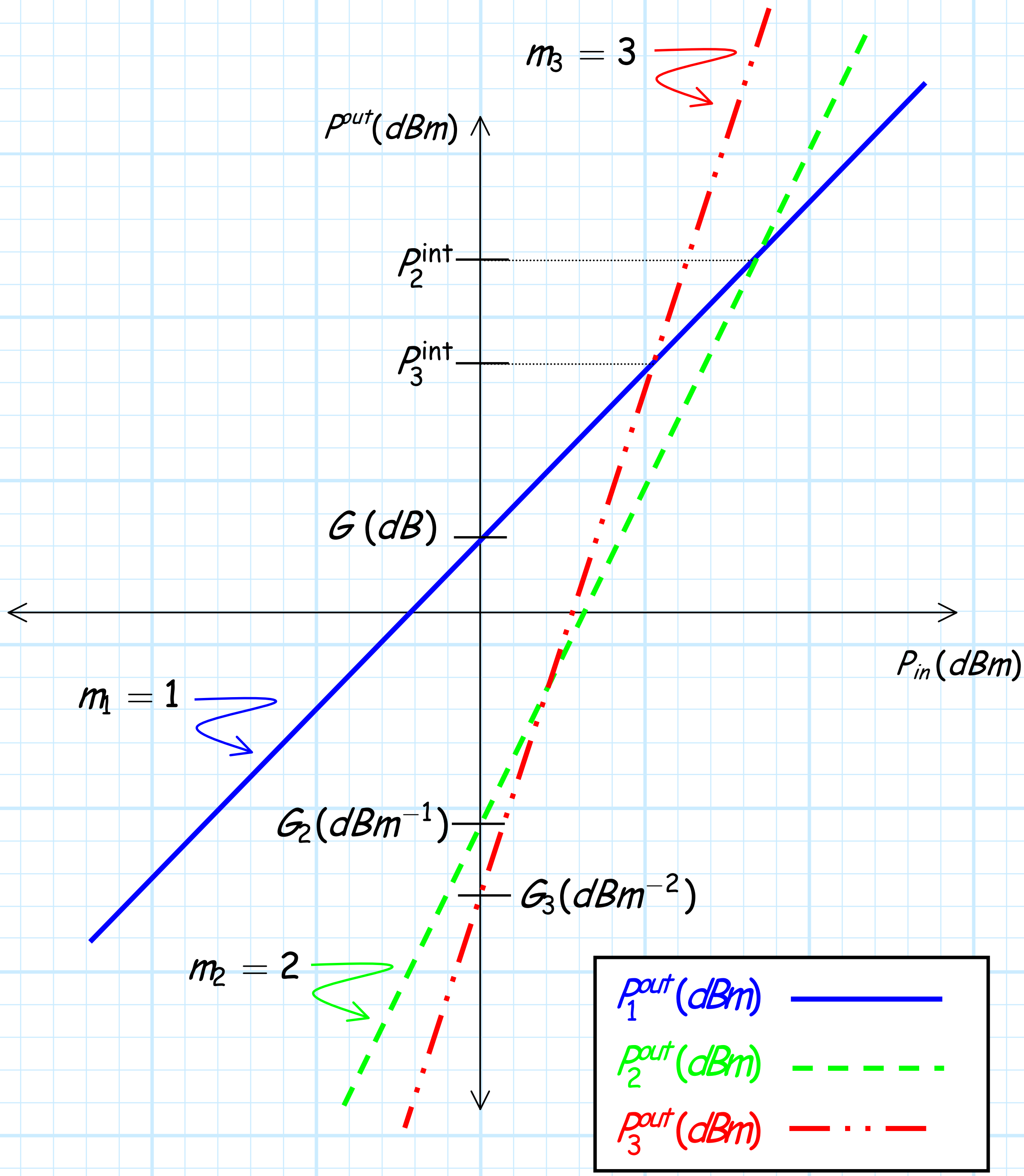
This is evident when we look at the three power equations (in decibels), as each is an equation of a **line** (i.e., $y = m\,x + b$).

For example, the equation:

$$P_3^{out}(dBm) = 3\left[P_{in}(dBm)\right] + G_3(dBm^{-2})$$
$$y = m\,x + b$$

describes a line with **slope** $m = 3$ and "y **intercept**" $b = G_3(dBm^{-2})$ (where $x = P_{in}(dBm)$ and $y = P^{out}(dBm)$).

Plotting each of the three equations for a typical amplifier, we would get something that looks like this:



Note that for $P_{in}(dBm) < 0\,dBm$ (the left side of the plot), the second and third-order products are small compared to the fundamental (first-order) signal.

However, when the input power increases **beyond** 0 dBm (the right side of the plot), the second and third order products rapidly **catch up**! In fact, they will (theoretically) become **equal** to the first order product at some large input power.

The point at which each higher order product **equals** the first-order signal is defined as the **intercept point**. Thus, we define the **second order intercept** point as the output power **when**:

$$P_2^{out} = P_1^{out} \doteq P_2^{\text{int}} \quad \textit{Second - order intercept power}$$

Likewise, the **third order intercept** point is defined as the third-order output power **when**:

$$P_3^{out} = P_1^{out} \doteq P_3^{\text{int}} \quad \textit{Third - order intercept power}$$

Using a little algebra **you** can show that:

$$\boxed{P_2^{\text{int}} = \frac{G^2}{G_2} \quad \text{and} \quad P_3^{\text{int}} = \sqrt{\frac{G^3}{G_3}}}$$

Or, expressed in **decibels**:

$$P_2^{\text{int}}(dBm) = 2\,G(dB) - G_2(dBm^{-1})$$

$$P_3^{\text{int}}(dBm) = \frac{3\,G(dB) - G_3(dBm^{-2})}{2}$$

* Radio engineers specify the intermodulation distortion performance of a specific amplifier in terms of the **intercept points**, rather than values G_2 and G_3.

* Generally, only the **third-order** intercept point is provided by amplifier manufactures (we'll see why later).

* **Typical** values of P_3^{int} for a **small-signal** amplifier range from +20 dBm to +50 dBm

* Note that as G_2 and G_3 **decrease**, the intercept points **increase**.

Therefore, the **higher** the intercept point of an amplifier, the better the amplifier !

One other **important point**: the **intercept points** for most amplifiers are much **larger** than the **compression point**! I.E.,:

$$P^{\text{int}} > P_{1dB}$$

In other words the intercept points are "theoretical", in that we can **never**, in fact, increase the input power to the point that the higher order signals are **equal** to the fundamental signal power.

All signals, including the higher order signals, have a **maximum** limit that is determined by the amplifier **power supply**.